Noise reduction in ventilation ducts

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Abstract

In this paper, the control of sound propagation in an air-handling duct using a feedback A.N.C. (Active Noise Control) system is investigated both from a theoretical and from a practical point of view. Using identified discrete time models of the duct, robust linear controllers are designed by means of the combined pole placement/sensitivity function shaping method. The specific features of the harmonic sources of noise and the study of the effect of model uncertainty on the A.N.C. performance are taken into account in order to avoid the classical stochastic adaptive and the reference noise based methods. As a consequence, the solutions proposed here are easier to implement in real time applications. As an illustration, we give detailed results for the control of the noise in an industrial air conditioner propagating through a duct.

I. Introduction

For over than 50 years active noise control techniques have been widely investigated as a relevant and complementary approach of passive noise techniques, but mostly on the theoretical point of view [1], [2].

Only recently high-speed processors have allowed the application of these techniques. The most widely used approach to A.N. C. is based on feedforward algorithms. The principle is to estimate the disturbance noise with a F.I.R. (finite impulse response) digital filter and to use a gradient-type algorithm to adjust the controller gains in order to reject the undesirable disturbances.[3],[4],[5] and [6].

This stochastic and adaptive approach provides efficient solutions to the so-called broadband problem which attempts to reduce a wide band noise to low frequencies. Moreover, some very interesting applications are also carried out. See for example [7] and [8]. Correspondingly, in periodic and narrow band problems, the LMS (least mean square) approach seems not to be the best candidate. In fact the implementation of the adaptive approach remains heavy, mostly because of the necessity to generate a filtered reference signal for every noise source, and the use of a stochastic framework that is not necessary in a large class of industrial problems where important prior knowledge is available.

An alternative approach to A.N.C. problem is to apply standard feedback techniques. State-space models have been widely used in active vibration control problems [9],[10], and more recently in active noise control scheme [11],[12],[13],[14]. The linear quadratic Gaussian (LQG) approach has been investigated by Wu et al. (see[13] and [14]) using a one dimensional state space representation of the process. The distributed parameter system is approximated by a finite dimensional model using a standard discretization technique with special boundary treatments. Hull et al. (see [11] and [12]) have developed a state-space model for a duct of a finite length and dissipative end conditions, and have achieved global noise reduction using the pole placement technique.

Recently (see [15]) an original and relevant augmented state-space model of the plant classically used in the $H_\infty$ control theory (see [16] for example) is presented. An analytical model is first derived. Then the parameters are adjusted experimentally using a least-squares method. An $H_2$ optimization method leads to high order controllers. Finally standard balancing and truncation techniques are used to reduce the order of the controllers that are implemented on a dSpace board leading to a noise reduction of 5 to 12 dB obtained over a bandwidth of about 200Hz.

As the authors noticed it, peaks of extra noise appear beyond the controlled frequency interval due to the classical trade-off managing the sensitivity function of the loop [17]. They also noticed the interest of applying direct robust reduced-order controller synthesis.

In this paper we propose to extend this analysis in the same direction while rectifying some inconsistencies [18]. Our first motivation is based upon the complexity of the phenomenon: the non-linearity of the process, the different types of wave fronts propagating through the duct, the acoustical interface at the open end of the duct too much roughly modelled so as the reflective waves involved, the fluid-structure interaction nearby the walls of the duct often neglected, the succinct modelling of the actuators and the sensors, the stock and theoretical assumption of a white noise modelling the disturbances often too far away from the reality, etc ... All these facts involved analytical modelling

1A full paper published in IEEE Trans. on Contr. Syst. Techn. (see [16]) presents in detail this work.
somewhat untrustworthy. It is rather significant to notice that analytical modelling has to be completed systematically by experimental adjustment.

Correspondingly, in the control community important improvements have been made recently providing efficient and reliable identification methodologies [19], [20], [21], [22] and [23], taking into account most of the previous characteristics mentioned above. Our second motivation is based on the fact that in a large class of industrial applications, the important prior knowledge of the physical disturbance characteristics are not taken into account and, as a consequence, the control methodologies remain global approaches as in [15]. Therefore the order of the controller thus obtained becomes excessive since the order reduction methods remain very delicate to apply. Here again, these last years important improvements in robust control methodologies have been made [24], [25].

The solution presented here used a digital robust controller combining the pole placement approach and the shaping of the sensitivity functions technique. This method is a part of the $H_\infty$ folklore. Still, it differs from the standard $H_\infty$ approach because of the way to shape the sensitivity functions. See [24] and [26].

This paper is organized as follows. In Section II, the air-handling duct is described in detail. Section III presents the discrete-time models associated with the duct. The robust control formulation of the problem is presented in Section IV while Section V is dedicated to the presentation of the control design methodology. Section VI deals with the acoustic plant identification and it gives the models thus obtained. The controller synthesis and the results of the real-time control of the system are presented in Section VII. Our conclusions are summarized in section VIII.

II. EXPERIMENTAL SETUP

A view of the experimental setup is given in Fig. 1.

![Fig. 1. Experimental setup.](image)

A. The Air-Handling Duct Description

We study the propagation of acoustic waves in a 3.40 m long, 0.20 m wide and 2.0 cm thick duct made of Plexiglas. Several tens centimeters of rockwool make one end of the duct almost anechoic. The other end of the duct is opened as it is often the case.

B. The Noise Sources

Kiel loudspeakers with 28cm x 18cm plane diaphragms are used to transmit the primary (disturbance) and the secondary (control) noise sources. The disturbance noise source may be achieved by a Tektronix model CFG 280 wave generator or, for real application, by the output signal of a microphone sensing the noise of an industrial aerotherm or air conditioner. See Fig. 8 and 9 below. Bruel & Kjaer model 2706 power amplifiers are used between the noise sources and the speakers.
C. The Sensor

The only microphone used in the feedback approach is a Bruel & Kjaer model 4130 microphone. This unidirectional microphone is devoted to the residual error signal, and is placed in the middle of the inner side of the duct, perpendicularly to its axis. The microphone is mounted with a Bruel & Kjaer model 2842 preamplifier providing initial amplification of the measurement signal, followed by a Bruel & Kjaer model 2810 power amplifier.

D. The Real-Time Control Tool

A complete package based on the D'Space and Mathworks products is used to achieve all the stages of the design and the real-time implementation of the robust control system. Simulink, Matlab and some of its toolboxes are chosen to offer a set of friendly and powerful tools for the simulation, the analysis and the design of the robust control systems. On the other hand, the D'Space software provides a convenient interface between the Mathworks products and the Texas Instrument DSP board DS1102 which we used in order to facilitate the implementation and the supervision of the real-time control systems. Therefore, as soon as the control algorithms are downloaded on the DSP board, the real time control of the process is assumed.

III. Discrete-Time Model for the System

The propagation of sound in the duct can be considered as an infinite dimensional process. Previous experimental work and theoretical analysis using either analytic modelling or finite elements modelling [13], [27], have shown that such a device has a very large number (theoretically infinite) of low damped oscillatory modes. In these delicate conditions semi-physical insight onto the process remains an interesting and efficient first step in the identification process. In this the energy is essentially concentrated on the first few modes. Therefore, in order to control the process, it is generally enough to control it in a range of frequencies covering the main modes. An upfront spectral analysis is usually sufficient to determine how many modes we have to take into account in a given application. Once the upper mode threshold has been determined, the sampling frequency $f_s$ of the discrete-time control model is chosen so that the Shannon frequency $f_{Sh} = 0.5 f_s$ lies between this upper mode and the next one. Then, appropriate anti-aliasing filters are applied, all this in order to make sure that the experimental data is enough to estimate the discrete-time control model representing the process.

The input signal of the system is applied to the amplifier of the control loudspeaker and the output signal is produced by the power amplifier of the error microphone. The model of the secondary path transfer is chosen in such a way to have a model structure of the form:

$$ G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} \quad (1) $$

where $d$ is the number of sampling periods ($T_s$) contained in the plant pure time delay, and

$$ A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_{n_A} z^{-n_A} \quad (2) $$

$$ B(z^{-1}) = b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{n_B} z^{-n_B} \quad (3) $$

with $n_A = 2N$ where $N$ is the highest acoustic mode considered.

IV. Control Problem Formulation

We now propose a formalism for the class of problems we are interested in. In the introduction, we already stressed that our approach was deterministic because the control problem itself is deterministic: we want to minimize the noise propagating in a duct whose the spectrum is the superposition of the spectrum of a low level background noise and peaks with unacceptable levels at specific frequencies. The latter are typically harmonics and sub-harmonics of a basic frequency related to the behavior of the noise source (e.g. a fan, an engine, ...). Unfortunately, the operating conditions involve in practice, some variations of these frequencies around their expected (or nominal) values, with a relatively weak range known only experimentally. Another serious problem is due to the working conditions which often

\(^2\)Simulink, Matlab and its toolboxes are products of the Mathworks Inc.
change (the variations of the propagation medium, the configuration of the free end, ...). Moreover, the kind of plant we study is usually modelled with an infinite order analytical models and of course, the numerical methods used to resolve them are of very high order. We classically used truncated models given by parametric identification algorithms. In this way, we obviously introduced an unstructured modelling uncertainty. Also, the unsteady state behavior of this kind of process leads us to take into account another kind of uncertainty due to, among other things, the variations of the frequencies to be rejected, the propagative medium and the limit conditions on the open air changes.

Classically, a robust control problem can be defined as follows. Is there any controller $K$ belonging to a given set of allowable controllers $\mathcal{K}$, such that, for an element $G$ of the given class of plants $\mathcal{G}$, the set of performances required $P$ belongs to a set of allowable performances $\mathcal{P}$?

In particular, we have to define the sets $\mathcal{K}$, $\mathcal{G}$ and $\mathcal{P}$.

**A. The Class of Uncertain Systems**

The standard noise levels allow us to work within the realm of the linear acoustic hypothesis. Note that in this framework, the zero value applied to the input of the loop means that the control problem is in fact a regulation one. Thus, we propose the control system block diagram shown in Fig. 2, where $G(z^{-1})$ denotes the plant, also called the secondary path (i.e. between the control source noise input and the measurement microphone output) around which we want to minimize the acoustic pressure.

![Fig. 2. Regulation of uncertain system.](image)

Under these conditions, we can say that the writing $\{G(z) + \epsilon\}$ represents in a general way the disturbed plant. Furthermore, the plant uncertainty, which is denoted by $\Delta G$, is due to the modelling errors and/or to time-varying physical parameters such as the propagation time delay or the propagation medium changes such as those observed at the open end of the duct. Therefore $G$ denoting hereafter the nominal plant, we define the set of uncertain systems labelled $\mathcal{G}_\epsilon$, or more simply $\mathcal{G}$, in a general way by:

$$\{G + \Delta G : \|\Delta G\| < \epsilon\}$$

(4)

where $\|x\|$ is a certain norm of $\Delta G$ defined on the functional space to which $G$ belongs. In the robust control theory, classically one uses the $H_2$ or $H_{\infty}$ norms. $\epsilon$ is a real positive number defining the subspace related to $\mathcal{G}_\epsilon$.

**B. The Class of Required Performances**

The required performances $P$ have two types:

1. those representing directly the user’s requirements, such as the primary noise rejection with specified level of attenuation (typically, some tens dB), eventually taking into account some variations of their values. For this reason, we define the nominal performance objective as the rejection of disturbances for the nominal plant only, and the robust performance objective when this rejection is guaranteed on the whole set $\mathcal{G}_\epsilon$.

2. the loop existence indeed implies also the guarantee of the loop stability for every $G$ belonging to $\mathcal{G}_\epsilon$. In terms of robust control theory, we name that as a robust stability objective in order to ensure the loop stability for the whole set $\mathcal{G}_\epsilon$.

**C. The Class of Allowable Controllers**

We have chosen a pole placement controller using an R-S-T regulator [28] even though, in a propagative process, predictive controller are generally used. The major motivation for using this approach is the possibility to easily solve control problems expressed in terms of specified frequencies. In any case, such a controller implicitly includes a predictor
and consequently, one should establish the equivalence between the two types of controller. Furthermore, in order to improve the robustness of the controller, we chose, among the different robust pole placement methods, a control design based on a technique based on the shaping of the sensitivity functions. The main reason is the prior knowledge we have on the disturbances and the fact that they are naturally expressed in terms of frequency. Therefore, using the terminology defined above, each $P$ belonging to the set of allowable performances $\mathcal{P}$ can be related to a set of shaping templates of the unknown sensitivity function. Recall that we already stressed that our control approach was chosen mostly to be a right candidate for the problem of selective rejection of unwanted frequencies.

We began the study of the mixed problem nominal performance/robust stability, assuming that the unwanted frequencies are constant in the control problem which we call $P_1 \in \mathcal{P}$. The resulting controller is called $K_1 \in \mathcal{K}$. Then, we checked the behavior of our approach when the unwanted frequencies are known with an uncertainty around their nominal values. This situation corresponds to a mixed robust performance/robust stability problem which we call $P_2 \in \mathcal{P}$. The resulting controller is called $K_2 \in \mathcal{K}$.

We also check that our method is capable to provide a solution to the so-called broadband problem. This corresponds to a mixed robust performance/robust stability problem which we call $P_3 \in \mathcal{P}$, relating to a set of uncertain systems $\mathcal{G}$ larger than the previous one. The resulting controller is called $K_3 \in \mathcal{K}$.

V. Robust Control Design Method

A. The Pole Placement Approach

In Fig. 3, a digital R-S-T controller generates the plant input signal $u(t)$ with respect to the the plant output $y(t)$ and the desired plant output trajectory $y_m(t + d + 1)$ taking into account the integer part of the time delay $d$, according to the time-domain control law:

$$S(q^{-1})u(t) = T(q^{-1})y_m(t + d + 1) - R(q^{-1})y(t)$$

(5)

where $q^{-1}$ is the backward shift operator: $y(t-1) = q^{-1}y(t)$. $R$, $S$ and $T$ are polynomials in $q^{-1}$.

![Fig. 3. Regulation loop using a RST controller and the pole placement approach.](image)

The desired trajectory $y_m(t + d + 1)$ is stored in a file or generated from the reference signal $r(t)$ by a tracking reference model of the form:

$$H_m(q^{-1}) = \frac{B_m(q^{-1})}{A_m(q^{-1})}$$

(6)

$H(q^{-1})$ is assumed to be a minimal representation of the plant, i.e. the polynomials $B$ and $A$ are assumed to be coprime. Therefore, writing the closed-loop transfer function between $y_m(t + d + 1)$ and $y(t)$, one can see that the closed-loop poles are the roots of the polynomial:

$$P(q^{-1}) = P_D(q^{-1})P_F(q^{-1}) = A(q^{-1})S(q^{-1}) + q^{-d}B(q^{-1})R(q^{-1})$$

(7)

where $P_D(q^{-1})$ defines the dominant poles derived from the required performances in regulation, and $P_F(q^{-1})$ defines the filtering poles, also called the auxiliary poles, guaranteeing certain intended effects such as robustness improvement of the closed-loop behavior and/or actuator stress. In fact, see [24], the controller’s polynomials $S(q^{-1})$ and $R(q^{-1})$
contain the prespecified parts \( S_2(q^{-1}) \) and \( R_S(q^{-1}) \) and in order to achieve requirements such as a null steady-state error involving \( S_S(q^{-1}) \) contains the term \((1 - q^{-1})\). Finally, the controller’s equation becomes:

\[
P_D(q^{-1})P_F(q^{-1}) = A(q^{-1})S_S(q^{-1})S_I(q^{-1}) + q^{-d}B(q^{-1})R_S(q^{-1})R_I(q^{-1})
\]

(8)

where \( S_I(q^{-1}) \) and \( R_I(q^{-1}) \) are the unknown terms to be determined.

B. Sensitivity Functions and Robustness Margins

The behavior of the closed-loop system is strongly modified by the disturbances. Consequently, it is important to thoroughly analyze their effects, either upon the plant output signal, or upon the plant input signal. For this reason, the sensitivity functions play a leading role in the robust control theory either according to the influence of disturbances or even to the robustness analysis of the closed-loop system. See for example [25] and [16]. Classically two types of disturbances will be considered here: the output disturbance \( e(t) \) and the measurement noise \( b(t) \). According to the Fig.3, we have the expression of the output (respectively, the input, the noise) sensitivity function:

\[
S_{ye} = \frac{AS}{p} \quad S_{ue} = -\frac{AR}{p} \quad S_{yb} = -q^{-d}\frac{BR}{p}
\]

(9)

Remark: \( S_{ye} \) is usually denoted \( S \) and is called the principal sensitivity function. \( -S_{yb} \) corresponds to \( T \) the so-called complementary sensitivity function. Here, in order to avoid confusion with the polynomials \( R \), \( S \) and \( T \) of the controller \( K \), we shall go on using in this paper the expressions \( S_{ye}, S_{ue} \) and \( S_{yb} \).

In the design of robust controllers, the modulus margin \( \Delta M \) and the delay margin \( \Delta \tau \) are fundamental. The first one defines the radius of the circle centered in \([-1, 0]\) and tangent to the Nyquist plot of the open-loop transfer function \( G_{OL}(z^{-1}) \). We have:

\[
\Delta M = \frac{|1 + G_{OL}(e^{-i\omega_T})|_{\text{min}}}{\max} = \frac{|S_{ye}(e^{-i\omega_T})|_{\text{min}}}{\|S_{ye}\|_{\infty}}
\]

(10)

As a consequence, a decrease of \( |S_{ye}|_{\text{max}} \) for some frequency will lead to an increase of \( \Delta M \). Therefore, \( \Delta M \) is used to define the upper template for the output sensitivity function \( S_{ye} \) to be shaped. Furthermore, owing to the disk criterion for the stability of nonlinear systems, \( \Delta M \) gives a bound for the characteristics of the nonlinearities and the time-varying parameter elements tolerated in the closed-loop system.

In the case of a propagative phenomenon, it is highly recommended to provide for the time-delay uncertainties. As a consequence, the delay margin \( \Delta \tau \) becomes fundamental for our robust control design. The time delay margin is derived from the phase margin \( \Delta \Phi \) and is defined by:

\[
\Delta \tau = \frac{\Delta \Phi}{\omega_{cr}}
\]

(11)

where \( \omega_{cr} \) is the crossover frequency, i.e. the frequency where the Nyquist plot intersects the unit circle.\(^3\) Classically one uses the values \( \Delta M \approx -6dB \) and \( \Delta \tau \approx T_s \).

It has been shown, [29], that the respect of time delay margins forces the modulus of \( S_{ye}(z^{-1}) \) to stay inside an area defined by a lower template \( |W^{-1}(z^{-1})|_{\text{low}} \) and an upper template \( |W^{-1}(z^{-1})|_{\text{up}} \). See Fig.4. The expression \( W \) is related to the classical weighting function used in the \( H\infty \) approach. For example, in the usual case of one sampling period delay time margin, the preceding templates are:

\[
|W^{-1}(z^{-1})|_{\text{low}} = 1 - |1 - z^{-1}|^{-1}, \quad \text{and:} \quad |W^{-1}(z^{-1})|_{\text{up}} = 1 + |1 - z^{-1}|^{-1}
\]

In the robust control framework, the constraints upon \( \Delta M \) and \( \Delta \tau \) express the robust stability objective as explained above. In the same way, we have already seen that the nominal performances objective consists in rejecting frequencies belonging to the spectrum of the disturbances noise source. Despite the different kinds of problems encountered, the respect of the nominal performances makes some rejection peaks appear in the design of the output sensitivity function as shown in Fig.4.

\(^3\)More generally, if the Nyquist plot intersects the unit circle at several frequencies, the time delay margin is given by \( \Delta \tau = \min \frac{\Delta \Phi}{\omega_{cr}} \)
Without going into detail (see [24] and [29]), all the control constraints discussed above lead to a thorough choice of the polynomials $P_F$, $S_S$ and $R_S$ which govern the resolution of the controller's equation (8). Let us remember that our objective is to design robust controllers for the problems $P_i$ described above, in Section IV. C. The means available in the shaping technique chosen is the appropriate choice of the relevant roots of the polynomials $P_F$, $S_S$ and $R_S$ in order to fit the sensitivity function curve in the templates expressing the control constraints. One can say that these templates generate a kind of manifold on which the curve has to stay (see Fig.4). Recently (see [30]), a convex optimization procedure has been proposed leading to Matlab toolbox, namely Optreg, taking automatically into account the constraints mentioned above and therefore facilitating the controller design.

VI. IDENTIFICATION OF THE MODEL OF THE DUCT

As shown in Fig.1, we have to find discrete-time dynamical models of the plant including the secondary speaker, its amplifier, the acoustic secondary path, the measurement microphone and its amplifier. The research of these models has to take into account the existence of analytic and numerical models using finite element methods, supporting a strong prior knowledge of the process. First theoretical analysis gives the expression of the propagative modes. Their cut-off frequencies are given by the general expression:

$$f_{c,m,n} = \frac{c}{2} \sqrt{\frac{m^2}{l_y^2} + \frac{n^2}{l_z^2}}$$  \hspace{1cm} (12)

where $c$ is the phase speed of the acoustic waves, $l_y$ and $l_z$ are the length and the width of the rectangular duct section respectively. Therefore the first theoretical propagative modes of the plant have the following cut-off frequencies ($l_y = l_z = 0.20 m$):

$$f_{c,0,0} = 0 \hspace{1cm} f_{c,1,0} = 850 Hz \hspace{1cm} f_{c,1,1} = 1202 Hz$$
$$f_{c,2,0} = 1700 Hz \hspace{1cm} f_{c,2,1} = 1901 Hz \hspace{1cm} f_{c,2,2} = 2404 Hz$$  \hspace{1cm} (13)

One should notice that because the duct has a squared section, we have $f_{c,m,n} = f_{c,n,m}$. This means that, for a given frequency $f$ of the disturbance noise source, the acoustic pressure observed is the superposition of all the waves propagating in the duct related to the modes $(m,n)$ such that $f_{c,m,n} \leq f$. For example, the frequency $400 Hz$ is propagating with a plane wave corresponding to the first mode $(0,0)$. Instead, the frequency $1000 Hz$ is propagating with a wave's combination of the two first modes $(0,0)$ and $(1,0)$.

This phenomenon will be taken into account in the identified transfer function by weakly damped complex poles associated with a time delay corresponding to the propagation in the the first mode: the plane wave mode in a perfect medium, i.e. of an infinite dimension, without attenuation, etc..

Moreover, all the noise energy to be reduced lies in the low frequencies (see section VII for an industrial illustration), on the other hand, active noise control is more efficient than passive absorption in low frequencies (typically some hundred Hertz) as we already explained in the introduction. Consequently, the Shannon frequency $f_{Sh} = 0.5 f_c$ has been...
chosen between the second \((f^c_{1,0} = 850\,Hz)\) and the third \((f^c_{1,1} = 1202\,Hz)\) theoretical cut-off frequencies of the duct. So a sampling frequency \(f_s = 2\,kHz\) has been chosen corresponding to a Shannon frequency \(f_{Sh} = 1\,kHz\).

The input signal used was a low magnitude pseudo-random binary sequence (PRSB). In Fig.5 we can see this input signal together with the output signal given by the microphone used for identification.

As we would not specifically identify the noise model, we have chosen the class of output error model (see [19] for more details, where \(n_A, n_B\) and \(d\) denote classically the number of poles, zeroes and sampling periods included in the time delay of the process model: \(y(t) = q^{-d} \left( \frac{B(q)}{A(q)} \right) u(t) + e(t)\) and where \(q^{-1}\) denotes the backward shift operator \((y(t-1) = q^{-1}y(t))\).

The estimation of the order \(n = \sup(n_A, n_B + d)\) of the model is a crucial problem in system identification. In the literature, parametric as well as non-parametric methods have been proposed to solve this problem. Parametric methods are based on the minimization of some criterion including the prediction error of the final model. The main drawback is the necessity of passing twice through the data, the first time for parameter estimation, and the second time for prediction error computation. Non-parametric methods try to avoid this shortcoming. They mostly rely on the instrumental variable technique (I.V. methods). We have used these different approaches. The results thus obtained are in agreement. A statistical analysis of the estimation errors leads to a quasi-whiteness of the prediction error [23]. A comparison between the time responses (impulse, step response) of the identified model and those experimentally measured gives satisfactory results. As we have already explained, a modicum of care is needed to check the phase curve, the propagative modes and the reflected waves at the open end of the duct, i.e. the physics of the process. As an illustration Fig.6 shows the comparison between the frequency response of the identified model and the spectral estimated from the data.

In the same vein, we also used closed-loop identification algorithms. The interaction between system identification and robust control has seen important developments since the nineties. They were motivated, on one hand, by practical constraints (plant which have an integrator behavior or are unstable in open loops, maintenance of existing controllers without opening the loop, etc) as well as by the fact that plant identification in closed loop may provide better models for design purposes. New algorithms have been developed, and some of them have remarkable properties [20] and [21]. More precisely, in our case the disturbance rejection at the process output leads us to the \(r-CLOE\) structure. The excitation signal in there added to the reference signal at the loop input as shown in Fig. 7.

The identification criterion is then:

\[
\hat{\theta}^* = \arg \min_{\theta} \left\| S_{yr} - \hat{S}_{yr} \right\|_2 = \arg \min_{\theta} \left\| S_{yp} - \hat{S}_{yp} \right\|_2
\]  

(14)
Fig. 7. r-CLOE: CLOE with external excitation added to the reference signal

See [31] for more details. The models thus obtained are very close to the ones previously found in open loop identification. Finally we have chosen the following Output Error Model:

\[
\begin{align*}
  d &= 6 \\
  A(q^{-1}) &= 1 - 1.3941q^{-1} - 0.0389q^{-2} \\
  &+ 1.2131q^{-3} - 1.1895q^{-4} + 0.0430q^{-5} \\
  &+ 1.0517q^{-6} - 0.6267q^{-7} \\
  B(q^{-1}) &= 0.0304q^{-1} + 0.0709q^{-2} - 0.0947q^{-3} \\
  &- 0.0170q^{-4} - 0.0104q^{-5} - 0.0787q^{-6} \\
  &+ 0.0414q^{-7} + 0.0380q^{-8} + 0.0250q^{-9} \\
  &+ 0.0366q^{-10} - 0.0584q^{-11} + 0.0540q^{-12} \\
  &- 0.0862q^{-13} - 0.6267q^{-14}
\end{align*}
\]

Fig. 8 gives the pole-zeros values of the identified model. There are zeros at infinity corresponding to the discretization of the propagation delay. One can see one unstable zero with a high absolute value which represents a time delay of almost one sampling period and pairs of unstable zeros corresponding to a pure nonminimum-phase behavior.

<table>
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<th>poles values</th>
<th>( f_\theta (\text{Hz}) )</th>
<th>( \xi )</th>
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<td>2, 3</td>
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<td>4, 5</td>
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<td>410</td>
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<tr>
<td>6, 7</td>
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<td>901</td>
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<table>
<thead>
<tr>
<th>zeros values</th>
<th>( f_\theta (\text{Hz}) )</th>
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Fig. 8. Poles and zeros of the identified model

VII. REAL TIME CONTROL OF THE DUCT

Our final objective being the application of this control approach in an industrial environment, we use the noise generated by an aerotherm, or air conditioner, propagating through the duct. In order not to complicate the initial problem we have deliberately chosen not to accommodate directly the aerotherm to the duct to avoid the delicate fluid
mechanics problem related to the mean flow. We focus here on the problem of the fan noise propagation through an open ended duct (see Fig.9).

Therefore we use a microphone stuck on the body of the aerotherm to sense the noise generated. Then an amplifier feeds the secondary or disturbance loudspeaker of the duct. One can see its low frequency spectrum in Fig.10 (solid line) made of a background noise mostly due to the air flowing from the the fan and propagating through the output blades of the aerotherm.

One can notice the presence of a peak at almost 130 Hz probably mainly due to the first vibration mode of the body of the aerotherm. This fact reveals that the control of a real process is much more complicated that the ones generally studied in simulation works, but nevertheless much more interesting. Due to instrumentation constraints the values shown in Fig.9 are ten times greater than the real values. One can also see peaks of sub-harmonic frequency (100 Hz) of the fundamental frequency (200 Hz) related the rotation speed and traducing in particular the non-linearity of the phenomenon. These facts reveal that the control of a real process is much more complicated that the ones generally studied in simulation works, but nevertheless much more interesting. We can notice that these two frequencies are the most important ones and that they belong to the frequencies particularly well heard by human ear. Such an indication is very important in real applications.

First a simulation analysis is done allowing the synthesis of the controller to be implemented. Fig.11 shows the block diagram of the dsp control board achieving the controller $K$ defined in Section IV. C, as it appears on the computer screen when using the Simulink software. Then the implementation of the real-time control algorithm is done and the supervision of the real-time control can start.

![Fig. 9. Aerotherm (noise source)](image)

![Fig. 10. Noise spectrum: with control (dashed line) and without (solid line).](image)

![Fig. 11. Block diagram of the closed-loop system under Simulink.](image)
A. The Mixed Nominal Performance - Robust Stability Problem

First, we have to reject as strongly as possible the undesirable peaks while ensuring satisfactory stability margin as explained above corresponding to the nominal performance problem, named $P_1$. According to the equations (7) and (8), we used: $P_F(q^{-1}) = (1 - 0.46q^{-1})^9$ as fast multiple real filtering poles and $P_D(q^{-1})$ including the seven natural identified modes, in order to improve the robustness of the system. In fact, we know that the last control modifies the regulation poles. $S_S(q^{-1})$ rejects the frequencies 100 and 200 Hz with a null damp, and two damped complex poles: $f_{3S} = 600 Hz / \zeta_{3S} = 0.015$, and $R_S(q^{-1}) = 1$. The order of the controller is 20 (the degree of the pre-assigned part $S_S(q^{-1})$ is already 6. Fig.12 (dot line) shows the $S_{yc}$ corresponding curve, related to the robust controller $K_1 \in \mathbb{C}$. Nevertheless, the disturbance signal $e$ is not experimentally available, only the output signal $y$ and the input signal of the primary (disturbance) noise source $p$ are measurable. Because $y$ and $e$ are approximately equals without control, the solution is to estimate the primary path transfer function between $p$ and $e \equiv y$ in these conditions. Therefore, it is possible to deduce the experimental transfer $S_{yc}$ to be compared with the theoretical used during the design.

However, in order to show the efficiency of the noise reduction, in Fig.12 (dashed line) we have drawn the curve: $20 \log \frac{|y_{nc}|}{|y_{nc}|}$ related to the frequency. In fact, this curve is the ratio between the output signals with $(y_{nc})$, without $(y_{nc})$ control. We can see the controller performances but we cannot early conclude about the respect of the stability margins. Finally, we have chosen this curve because it is near the user’s concerns rather than the designer’s ones. It is interesting to notice that the measurements at the peak frequencies are: -55 and -48 dB at 100 and 200 Hz. The result of the control in the frequency domain is also shown in Fig.9 where one can see the spectrum of the noise with (dashed line) and without control (dot line).

![Fig. 12. Experimental results - the rejection curve (controller $K_1$).](image)

B. The Mixed Robust Performance - Robust Stability Problem

Next, in order to take into account possible changes in the undesirable frequencies, we designed the controller $K_2$ using: $P_F(q^{-1}) = (1 - 0.46q^{-1})^9$ as previously explained, $P_D(q^{-1})$ including the seven identified natural poles, $S_S(q^{-1})$ is: $f_{1S} = 100 Hz / \zeta_{1S} = 0.04$, $f_{2S} = 200 Hz / \zeta_{2S} = 0.02$, $f_{3S} = 600 Hz / \zeta_{3S} = 0.015$ rejecting these three frequencies, and $R_S(q^{-1}) = 1$. Fig.13 shows results. The dot line shows the theoretical $S_{yc}$ curve established during the synthesis of the robust controller $K_2 \in \mathbb{C}$. In order to define the class of uncertain systems $\mathcal{G}$ considered here, we force the theoretical $S_{yc}$ curve to present around the nominal values of the frequencies to be rejected, namely 100Hz and 200Hz, a bandwidth of about ±5 measured at 3 dB from the lowest level of each rejection peak. The order of the controller thus obtained is still 20.

The solid line represents the experimental $S_{yc}$ curve. The results are very satisfactory. The measures show a rejection of about 17 and 19 dB for the unwanted frequencies with a tolerance of about ±10%.

The problem is now to reject the frequencies between 30 and 170 Hz. The third design $K_3$ is achieved using: $P_F(q^{-1}) = (1 - 0.48q^{-1})^{12}$ slightly different from the previous case. $P_D(q^{-1})$ including the first identified natural modes $f_{1Q_4} = 116 Hz / \zeta_{1S} = 0.1$ which are damped. $S_S(q^{-1})$ is: $f_{1S} = 50 Hz / \zeta_{1S} = 0.1$, $f_{2S} = 100 Hz / \zeta_{2S} = 0.1$, $f_{3S} = 150 Hz / \zeta_{3S} = 0.1$, $f_{4S} = 200 Hz / \zeta_{4S} = 0.1$, $f_{5S} = 250 Hz / \zeta_{5S} = 0.07$, $f_{6S} = 318 Hz / \zeta_{6S} = 0.06$ rejecting these six frequencies, and $R_S(q^{-1})$ has the form: $f_{1R} = 540 Hz / \zeta_{1R} = 0$, $f_{2R} = 730 Hz / \zeta_{2R} = 0$. The design becomes complicated because we have changed the natural dynamic of the system. Fig.14 shows the results. The order of the controller thus obtained is greater than 30.
Fig. 13. Experimental results - the rejection curve (controller $K_2$).

Fig. 14. Experimental results - the rejection curve (controller $K_3$).

The dot line shows the theoretical $S_{ye}$ curve established during the synthesis of the robust controller $K_3 \in \mathcal{K}$, obtained with difficulty, imposing many auxiliary poles and zeroes. The solid line represents the experimental curve. The results are average. Even though some frequencies are rejected with $13 \, dB$, one can notice that the rejection is not regular on the required frequency's range. An increase in noise is also noticed in the $150 - 170 \, Hz$ area.

VIII. Conclusions

In this paper we have proposed a noise control method especially designed for a large class of problems corresponding to the rejection of a finite number of undesirable frequencies with or without an uncertain margin. We successfully applied it to noise reduction in ventilation ducts. We have focused on a robust control methodology, avoiding the classical stochastic adaptive methods and the reference filtered signal in order to propose a solution much easier to implement in real time applications.

Concurrently, we noticed the importance of the phase response identification which involves a large number of zeroes in the transfer function thus obtained.

We proposed robust pole placement reduced-order controllers using the shaping of the sensitivity function leading to robust reduced-order controllers. It allows for the choice of the frequencies to be rejected, and to adjust the width of the rejected band. The stability robustness is guaranteed by the respect of the modulus and the delay margins.

The experimental results obtained in real time applications, strengthening and extending those obtained in simulation, prove the validity of our approach. The performances measured are quite interesting in narrow band problems: between -50 and -60 dB in the nominal problem and around -18 dB with a $\pm 10\%$ tolerance margin on the frequency to be rejected in robust performance problems. As expected, the results of this method are less impressive in the so-called broad band case where the right solution seems to be the classical feedforward approach despite its inherent intensiveness. Notice that in this case we are approaching the white noise case.

We conclude that the method proposed here appears to be a prime candidate to the solution of the narrow band problem, in terms of performances, but most of all, in terms of its simplicity of adjustment and implementation in practical applications. Indeed, it is quite easy to implement this kind of algorithms in a cheap DSP board leading to a low cost industrial solutions.

References